Power Generation from Aeroelastic Flutter at Low Reynolds Number

Design, Fabrication and Analysis of a Wind Belt System

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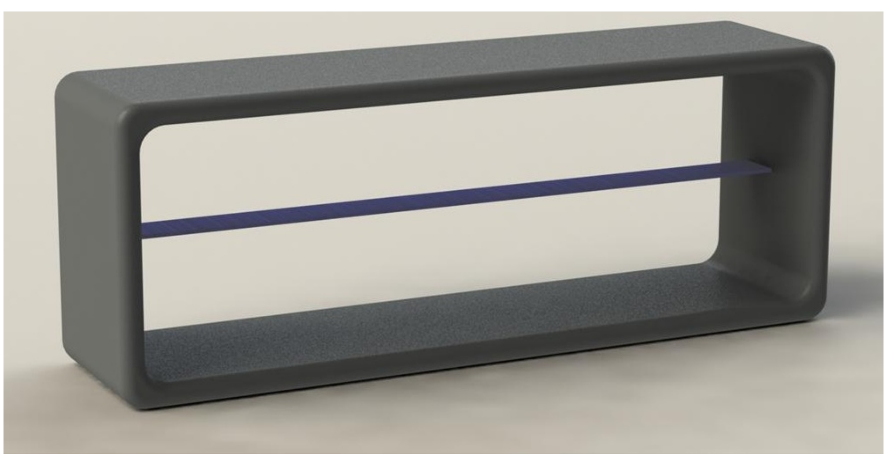
Introduction

The purpose of this initiative was to combine fluid mechanics and vibrations to create a device that generates electricity from wind energy. The phenomenon of aeroelastic flutter was used to produce electrical energy with the metric of success being power generation. This task was divided into two major components. The first phase was focused on designing an apparatus, prototyping it, and performing initial testing to gain preliminary insight into its functionality. The second phase of the effort was focused around deriving a mathematical model of the system. The model that was ultimately created uses simple equations of motion to determine the natural frequencies for each degree of freedom of the windbelt under varying configurations. In the third and final phase, magnet induction using copper coils and magnets was used to generate power from the oscillatios.

Ultimately, the windbelt was successful in the wind tunnel, generating 0.008 Watts of power at a wind-speed of 13 miles per hour. In addition to successfully generating enough power to light a light emitting diodes, this task provided an experiential learning experience in understanding system modeling, device creation and targeted design through engineering principles.

Description of Physical Device

After conducting extensive research, a wind belt was settled on as an incredibly efficient and simple method of generating electrical power. The tensile band within the system can be idealized as a flat plate that exhibits aeroelastic flutter ("Windbelt Notes", 2009). Magnets fixed to the band oscillate inside a coil, generating electrical power. The system consisted of a tensile band that was fixed at both ends by a rigid plastic casing. The apparatus' overall dimensions were 10 inches in width, 2 inches in length, and 3 inches in height. The belt spanned the entire width of the frame and had a chord length of 0.625 inches. It's thickness was on the order of thousandths of an inch. The coil consisted of 31 gauge wire, wrapped to a height of 1.15 inches and an external diameter of inches 1.75 inches. This resulted in approximatley 2000 coils. This was estimated by measuring the rotational speed of the drill and timing the spooling process. The inner diameter of the coil is 0.5 inches to accommodate a magnet stack of diameter 0.25 inches. The resistance of the coil was found to be 72 Ohms using a DMM. The apparatus is shown below.



Analysis

Design

Dr. Bruce Kothman was consulted to gain some insight into the use of airfoils (Kothmann, 2009). Dr Kothmann explained that a simple airfoil, with center-of gravity aft of both the airfoil's elastic axis and center of pressure, attached to a multi-spring system would create a reasonable amount of flutter for the production of power (Herbert et. al, 1996). Modeling the apparatus in SolidWorks uncovered inherent system flaws; specifically the linkage necessary to draw power from an oscillating airfoil would suffer massive frictional losses thus reducing power output dramatically (Tang et. al, 2009). These finding lead to a new design focus based on reducing frictional losses of the entire system. After further research, windbelts were settled on as an incredibly efficient and simple method of power generation.

Inititally, a highly elastic tensile band was installed in the windbelt's frame. While this material behaved ideally during intial testing, the additon of magnets and their associated weight yielded substantial performance losses. To remedy this shortcoming, various materials were tested to replace the elastic band. A mylar strip was ultiamtely found to be the most effective fluttering specimen in the magnetically loaded condition. What was required was that the magnets oscillate primarily in the vertical direction. With the mylar strip it was visually obvious that the vibrations were occurring strictly along a vertical axis. The mylar strip was a better candidate because of its stiffness to weight ratio. If the magnets make up most of the mass in the belt system then the oscillations are unstable; if the belt is not stiff enough the oscillations will also become unstable.

Several chord lengths were tested in the wind tunnel to determine the best configuration that resulted in the stable oscillations of a large amplitude. Testing showed that a chord length of 0.625 inches was most appropriate. The following table shows the trials with varying chord length and the formula below that shows the affect of the chord length on the oscillations.

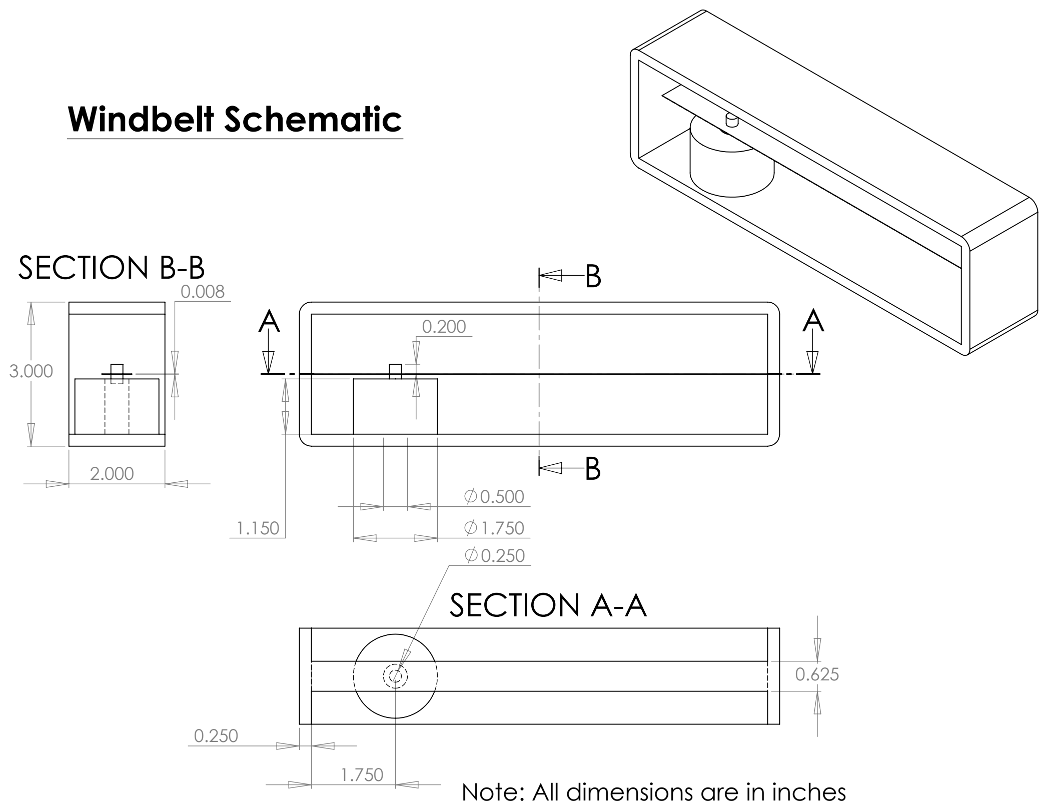
Trial Number	Chord Length (in.)
1	3/32
2	9/64
3	6/32
4	1

$$f_{torsional} = \frac{1}{l}\sqrt{\frac{K}{J}} \qquad J = \frac{m * b^2}{12}$$

In this equation b is the chord length which demonstrates the susbstantial affect it has on the torsional frequency. It has been shown below that stable oscillations rely mainly on the torsional and heaving frequencies to be related by an interger, since the chord length only afftect the torsional frequency this is something that had to be optimized.

Given the limited availability of magnets of a pre-defined strength, the belt's optimization relied on finding the best configuration of magnets. In addition to varying the number of magnets present, their location on the belt (both laterally and chord-wise) were variable parameters that were found to have substantial effects on the flutter behavior (see the vibrational analysis section for further discussion).

In order to take full advantage of the wind tunnel's available cross section (up to 25% of its total area), the initial windbelt was expanded using a simple expansion coupler to span the tunnel's entire width. By having a wider frame, the belt's length could increase, thereby resulting in oscillations of a greater amplitude. This was never modeled but was discovered empirically during testing. A schematic of the windbelt is shown below.



Vibrational Analysis

Dr. Michael Carchidi's help was taken in deriving the mathematical model for the system. Dr Carchidi explained that the model would require some simplifying assumptions similar to those used in modelling the Tacomas Narrows Bridge collapse. To model the belt's heaving motion, the system was simplified as a vibrating string and the torsional vibrations were modeled with a twisting rod. The model's parameters were the tension, chord length, aspect ratio, and overall elasticity (stiffness constant). Each of these factors play a key role in establishing the belt's natural frequency for both modes of vibration. In order to fully understand how each of these factors contribute to the belt's motion, a Matlab program was be used to calculate the equations of motion for the belt based on all of these variables and the complex interplay between them which is further explained in the analysis section.

Experimental Procedures

To verify the results from the mathematical model, the system was tested by perturbing the tensile band from equilibrium and measuring its vibrational frequencies using a phototachometer. Due to damping forces, simply 'twanging' the band was insufficient to allow for accurate measurements since all motion ceased in under a second. An alternative method of testing was employed wherein the system was placed in front of a fan, whose flow of air was both turbulent and irregular (similar in nature to repeated 'twanging'). Since tension is the driving factor behind the band's natural frequency of vibration, it was varied as measurements of both the heaving and torsional frequency were taken. Three trials were performed for each tensile value and the associated frequencies were averaged in order to reduce the likelihood of error.

Overview of Analysis

Differential analysis was used to determine the natural frequencies of oscillation for the system. The wind belt is a two degree of freedom system so decoupling was used to solve for each DOF frequency. Idealizing the band as a vibrating string in heaving motion, yielded simple solutions which matched very well with the data taken. The torsional vibrations were modeled using formulas derived for determining the motion of the famous Tacoma Narrows Bridge.[1]Secondary analysis was performed using emprical data from the wind tunnel during part one of the lab, as well as manually induced oscillations of the band. This was used to prove that the equations had yielded reasonable models of the system.

Vertical Vibrations

Modeling the vertical vibrations required some data to be taken on the system. First the tension was measured in the system using a hanging mass that stretched the belt to its proper length. The mass of the band was measured using a scale accurate to .1 grams. Finally the dimensions of the belt were meausured. The two latter quantities allowed the mass per unit length to be calculated.

Analyzing the vertical vibrations as a flexible string with mass per unit length ρ, tension T, length l and with boundary conditions $y(0,t) = y(l,t) = 0$ the general solution is:

$$\sin \tfrac{\omega l}{c} = 0$$

This is satisfied by

$$\tfrac{\omega l}{c} = \pi, 2\pi, 3\pi, \ldots, n\pi$$

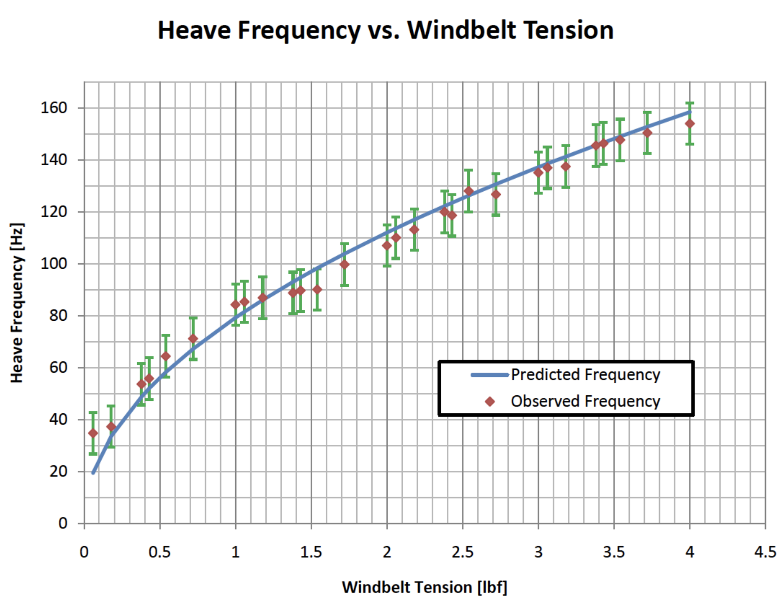
Which can be rewritten to determine the frequency, where n is the mode number:

$$f_n = \frac{n}{2l}\sqrt{\frac{T}{\rho}}$$

Using the following conditions $l = .5833ft$, $T = .37lbf$, and $\rho = 1.17 \times 10^{-4} lb \cdot ft^2 \cdot s^2$ and for $n = 1$

$$f = 48.1\,Hz$$

This result matched very well with data collected from testing:



[1]W.T. Thomson, "Vibration Periods at Tacoma Narrows," *Engineering News Record,* Vol. P477, pp.61-62 (March 27,1941)

Torsional Vibrations

Examining the natural frequencies of torsional vibration required a much higher level of analysis. The problem was again idealized as a twisting rod of length l which is constrained at both ends. Using principles of statics the polar mass moment of inertia can be related to the stiffness of the rod by the following equation of motion:

$$J\frac{\partial^2\theta}{\partial t^2} = K\frac{\partial^2\theta}{\partial x^2}$$

The solution to this equation is found by separating the functions where:

$$\theta(x,t) = f(x)g(t) \Longrightarrow Jfg^{''} = Kf^{''}g$$

$$\tfrac{J}{K}\tfrac{g^{''}}{g} = \tfrac{f^{''}}{f} = -\lambda^2$$

Solving for these two equations yields:

$$f^{''} + \lambda f = 0 \qquad g + \frac{k\lambda^2}{J}g = 0$$

For these equations:

$$\omega^2 = \tfrac{K}{J}\tfrac{\pi^2}{L^2} \qquad J = \tfrac{m*b^2}{12} \qquad K = T*b^2$$

In both equations, b is the chord length.
Plugging this in for the general solution to the equation:

$$\theta(x,t) = \left(A\sin\omega\sqrt{\frac{J}{K}}x + B\cos\omega\sqrt{\frac{J}{K}}x\right)(C\sin\omega t + D\cos\omega t)$$

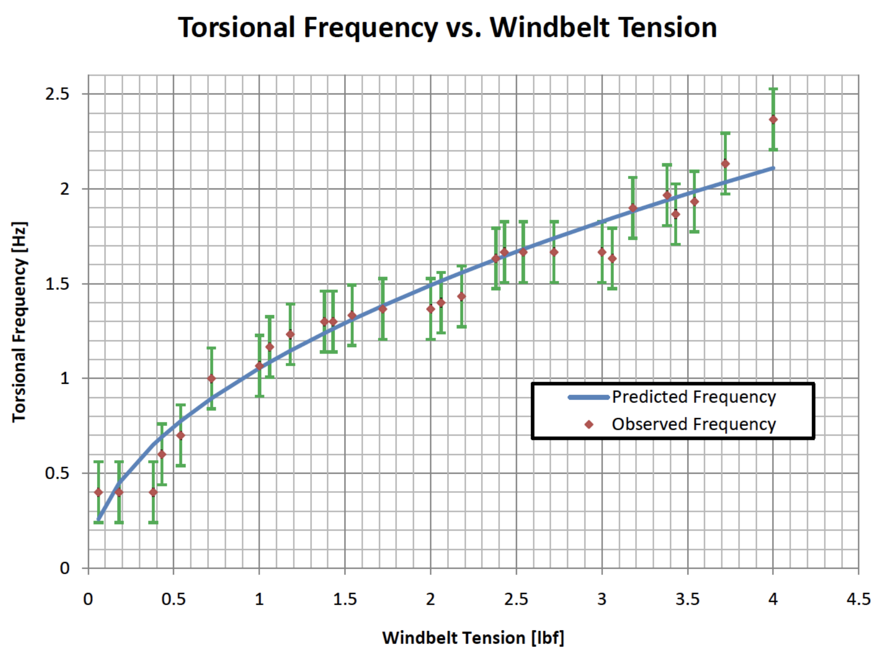
Solving for the natural frequency using $\theta(0,t) = \theta(l,t) = 0$

$$\sin\omega\sqrt{\tfrac{J}{K}}l = 0 \Longrightarrow \omega\sqrt{\tfrac{J}{K}}l = \pi, 2\pi, \ldots, n\pi$$

The observed oscillations were 2 node at the midspan, plugging this in for n the frequency is:

$$f = \frac{1}{l}\sqrt{\frac{K}{J}} = 38.5\,Hz$$

This frequency was hard to observe but using a phototachometer, data was obtained for varying tensions:



Interpretation

Given the physcial dynamics of wind belts, stable oscillations occur when the frequency of heaving oscillations match that of the torsional oscillations. Using this type of analysis it becomes quite clear that the calculations yielded reasonable answers. The frequency of heave was $48.1\,Hz$ and in torsion it was $38.5\,Hz$. These are reasnably close given the errors that naturally occur when idealizing a system such as this. Specifically, the band is not a rod and thus will behave slightly differently, combined with the fact that in reality these two degrees are coupled together. Finally this analysis doesn't make use of any forcing function, like that of the wind against the band which obviously has an effect on the length of the band which will change the vibrational tendencies.

The testing conducted showed that the mathematical model was extremely accurate in predicting the vibrational frequency of oscillations. Errors in the data come from many sources, some which are unquantifieable like friction at the attachment points and inconsistencies in the belt itself. One error that can be analyzed was the error in the heaving frequency that was observed at higher tension. As the belt is tensioned more the chord length decreases and thus the mass per unit length decreases, which would yield lower frequencies than the calculations predict. Secondly, aerodynamic effects play a large roll for low tension and thus the oscillations will be more choatic and tend to be slightly higher. Finally, analyzing error in the torisional vibration is difficult given how complicated the coupling is between torsion and heaving in the belt.

A final large factor in determining the vibrational tendencies of the windbelt is the addition of magnets to the belt which will be used to produce current. Adding unbalanced weight to one side will most likely create a 2 node oscillation in heaving which will increase the frequency by a factor of two but change the dynamics of the oscillations i.e. the amplitude and force. More testing must be completed before decent estimates of the power output can be derived however there is a stable jumping off point derived from the current calculations.

Electormagnetic Induction

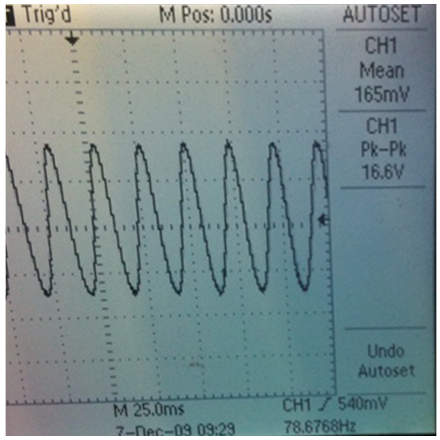
Testing in the wind tunnel was conducted at a constant airpseed of 13mph this resulted in a Reynolds number of approximately 655, corresponding to flow in the laminar regime where viscous forces dominate. $Re = \frac{\rho V L}{\mu}$

This result was surprising, however the empirical testing of belts of different chord lengths supported this conclusion. The resistance of the coil was used to optimize the resistance of the ciruit that would be needed. The differential calculations yeieled a resistance of 72 Ohms, the same value as the coil. Maximum useful power occurs when:

$$\frac{dP_{useful}}{dR_d} = 0 \qquad dP_{useful} = \frac{V^2 R_d}{(R_d + R_c)^2}$$

$$\frac{dP_{useful}}{dR_d} = \frac{V^2}{(R_d + R_c)^2} - \frac{2V^2 R_d}{(R_d + R_c)^3} \qquad R_d + R_c = 2R_d \therefore R_d = R_c$$

Initial testing was performed using this resistnace, however when a 10 Ohm resistor was used the damping from the increase in current and thus magnetic field stregth, was negligible and the power output greatly increase. In the end a 10 Ohm resistor was used in the circuit, producing a peak-to-peak voltage of 0.8 milliVolts generated, with a root-mean-squared voltage of 0.2828 Volts. Thus, the success metric of power generation was met when 0.008 Watts of power were generated, enough to power an LED. This power output was consistent with the expected results. The image below is a picture of the wave form produced by the coils.



Field Strength

Calculation of the magnetic field strength was derived from class notes and is shown below.

$$\mid B \mid = \frac{\mu_0}{4\pi} 3\mu \frac{hr}{(r^2+h^2)^{\frac{5}{2}}}$$

Using $\mu = \frac{4}{3}\pi r^3 * (1/\mu_0)$

$$\mid B \mid = \pi^2 r^3 \frac{hr}{(r^2+h^2)^{\frac{5}{2}}} = 2.65T$$

This result seems reasonable, our coil is producing a magnetic field about 2.5 times the strength of our magnet.

$$\frac{d\phi}{dt} = \frac{\varepsilon}{-n} = \frac{.0008}{2000} = 4x10^{-7} webers/s$$

This is the magetic flux over time of the system.

Power Efficiency

The power efficiency of our system can be measured in three ways:

1) Energy from the wind coming into the system, $P = F * V$
2) Energy from the mechanical system, $KE = \frac{1}{2} m_{belt} V_{belt}^2 + \frac{1}{2} m_{magnet} V_{magnet}^2$
3) Energy from the circuit, $P = I^2 R = \frac{V^2}{R}$

Discussion

With every additional testing session, it was noted that the windbelt's performance diminished. That is, both the amplitude and frequency of its oscillations steadily decreased as the belt experienced wear, resulting in decreased power generation over time. Future designs based of of this would would have to be made in a more robust fashion to prevent this performance degeneration from occurring. To that end, perhaps the belt can be coated in a resilient material on one side to prevent permanent deformations.

Another modification that would yield substantial performance improvements would be the refinement of the wire coils. If the magnet's motion could be further linearized, the inner diameter of the coils could be reduced. Since the power generation varies exponentially with the distance between the magnets and the coils, this would result in substantial performance improvements.

Sources of Error

In order for the post-analysis of the windbelt system to be complete errors must be taken into account. Firstly, as with most mechanical systems, stress and strain play a large role in overall stability of the mechanism. The windbelt was especially sensitive to small changes in belt tension and boundary conditions and therefore the wear and tear of the system was a huge source of error because it was almost impossible to quantify. One main problem we noticed was that as the belt was tested more and more it became noticeably fatigued and the oscillations changed and became more unstable. This makes sense because the belt relies on homogeneity to oscillate stably. A second source of error arose from the fact that the apparatus was moved in and out of the wind tunnel several times. This made it impossible to verify if the positioning on the belt was consistent, which brings up another source of error, the wind tunnel itself. If the windtunnel was idea, the flow would be laminar and constant across the entire testing cube. Variations in windspeed have a sizable affect on the windbelt's performance and the distance of the windbelt to the fan has an impact on the force of the wind as well. The speed of the wind which was measured by pressure sensors was fairly accurate, however the display was calibrated by hand and therefore there was a large error margin there as well.

Finally, there were multiple sources of error that arose from power generation. Firstly, the magnetic damping that occurs was difficult to model, not because the force was difficult to measure, it is a simple voltage calculation, but because the location of the force had a massive impact on the vibrations and the distance was difficult if not impossible to measure. Lastly, miniscule changes to the location of the magnets and the coils had a huge affect on the system. These changes were almost impossible to quantify and turned into a guess and check scenario.

Overall the errors in the system were large, making a model difficult to create that would behave accurately, however the analysis sheds light on which areas to focus attention on in order to make a more stable system, the belt strength and magnet positions.

Conclusions

During final experimental tests, the model of the wind-belt produced 0.008 Watts of power, thus the primary metric of success was achieved . Consistent high frequencies of 53 Hz from wind-speeds of 13 mph allowed for the magnet-coil setup to produce this power steadily. In fact enough power to light an LED was generated by the windbelt. This success was made possible in part through the accurate vibrational calculations analysis of the vibrating belt under the circumstances encountered by the belt.

Retrospective

Geoff Johnson

I was excited at the overall prospect of this lab because it encorporates a lot of aspects of engineerings to accomplish a huge need, energy creation. I was a little frustrated to learn that we were being restiricted in our solutions range but was still excited to take on the challenge of desiging a fluttering airfoil. It became apparent very quickly however, that this was not the most efficient power generation method and that a wind belt would be better. I really enjoyed learning about wind belts and how the system worked both dynamically and electrically. This lab definitely gave me a greater understanding of mechanical vibrations and power generation, which is something that I consider to be paramount for a lab.

Lucas Hartman

Tejus Goenka

At the start of this lab, our group was very excited by the wide range of possible solutions to the non-traditional task at hand: generating wind energy from flutter. We were slightly disheartened when it became apparent that there were only two practical solutions to choose from: windbelts and airfoils. However, as we progressed through the lab, it became more and more interesting to analyze a complicated system from a dynamic standpoint using fluid dynamics and vibrational models. This was the first experience we have had in engineering a system that could be applicable to the real world, hence our excitement grew as t he obtained results matched the predicted theoretical ones.

However, we became frustrated when we realized that the initial windbelt material that we had selected was not ideal. Obtaining reliable data for the substitute belt proved to be quite difficult and thus we resorted to a more rudimentary approach of guess-and-check to correctly calibrate our system. Our previous vibrational model provided us with comprehensive physical intuition for the system allowing us to calibrate our system well using a trial-and-error method.

Physically wrapping the coils was not difficult, however optimization proved to be a more difficult task. Even with reasonably sufficient knowledge of electromagnetic induction, it was difficult to physically create well-structured coils per the desired specifications was a major challenge.

In the final steps of the lab, we were pleased with our final results and pleasantly surprised by the power generation that the windbelt had achieved. The belt's oscillations were stable and the waveform readings obtained from the oscilloscope verified that these stable oscillations were generating a significant amount of power. However, we were again frustrated when we returned to the wind tunnel to perform some post-testing for the lab report's analysis to find that the widnbelt had fatigued slightly resulting in unstable oscillations.